THE EFFECT OF THE COMPRESSION OF WAVY SURFACES ON CONTACT THERMAL RESISTANCE

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In this article we obtain expressions which relate the contact thermal resistance between surfaces with spherical and cylindrical undulations to the external load imposed on them. It is assumed that the amplitudes of the waves obey a Gaussian distribution and that the microscopic asperities have constant radius.

In earlier papers relating to microscopic asperities of constant [1] and variable radius [2], we derived analytic expressions relating the value of the contact thermal resistance between rough surfaces to the external load imposed on them. In the present paper we shall find an analogous relationship for surfaces with spherical and cylindrical undulations. According to [3], these surfaces can be simulated by a set of corresponding waves of constant radius, their heights above a certain reference plane M (Fig. 1) obeying a Gaussian distribution. The radius of the microscopic asperities may be considered constant.

As in [1], instead of considering contact between two similar undulating surfaces, we consider contact between a smooth surface N and an undulating surface P with a certain "effective" dispersion of the amplitude distribution. For a distance d between the planes M and N, the number of waves which come into contact with the smooth surface will then be equal to

$$n_i = \frac{N_i}{\sigma_i \sqrt{2\pi}} \int\limits_d^{C^*+r_0} \exp\left(-\frac{C^2}{2\sigma_i^2}\right) dC = \frac{N_i}{2} \operatorname{erfc}\left(\frac{d}{\sqrt{2}\,\sigma_i}\right), \tag{1}$$

$$\operatorname{erfc} x = 1 - \Phi(x),$$

where $\Phi(x)$ is the probability integral [4]. In what follows the subscript $i = 1$ will refer to spherical undulations and the subscript $i = 2$ to cylindrical undulations.

Spherical Undulations. According to [5], for undulating surfaces the total contact thermal resistance consists of two components. One is due to the contraction of the lines of thermal flux to a fair-sized region of concentration of discrete contact spots, which has the shape of a circle for spherical undulation and a strip for cylindrical undulation. The second component is due to the direct contraction of the lines of thermal flux to individual contact spots. According to the recommendations of [6], the radius of the region from which the lines of thermal flux contract to an individual contact spot may be taken equal to $2r_0$. For A we find that

$$A = \sqrt{(R+r_0)^2 - (R+r_0-B_j-2r_0)^2} \approx \sqrt{2(R+r_0)(B_j+2r_0)}. \tag{2}$$

From geometric considerations it follows that the number of microscopic projections of an individual wave having a deformation b_j $(0 \le b_j \le B_j)$ is

$$n_1(b_j)\,db_j = \frac{\pi(R+r_0)}{8r_0^2}\,db_j, \tag{3}$$

and the integrated conductivity through the discrete contact spots of an individual wave will therefore be

$$\alpha_1 = \pi\lambda \int\limits_0^{B_j} \frac{a}{\operatorname{arctg}\dfrac{2r_0-a}{a}}\, n_1(b_j)\,db_j = \frac{\pi}{6}\lambda(R+r_0)\left(\frac{B_j}{r_0}\right)^{3/2}, \quad a = \sqrt{r_0 b_j}, \tag{4}$$

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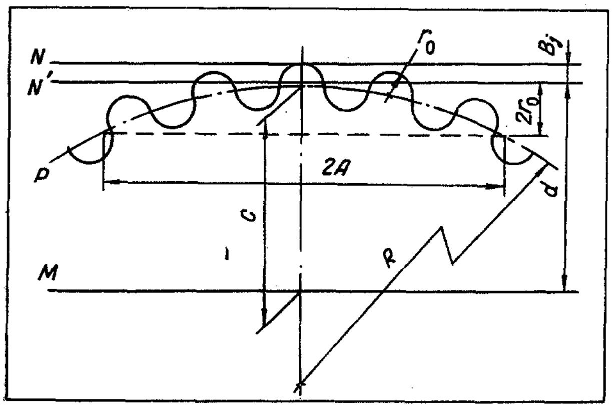

Fig. 1. Model of wavy surface.

and its total resistance to heat flow may be written in the form

$$R_1 = \frac{1}{\pi\lambda A} \operatorname{arctg} \frac{r_1 - A}{A} + \frac{1}{\alpha_1}, \quad r_1 = \frac{1}{2}\sqrt{\frac{1}{n_1}}. \tag{5}$$

The conductivity of unit area of the nominal contact surface is the integrated sum of the conductivities through the individual waves, i.e.,

$$\alpha_1^* = \frac{N_1}{\sigma_1\sqrt{2\pi}} \int\limits_d^{C^*+r_0} R_1^{-1} \exp\left(-\frac{C^2}{2\sigma_1^2}\right) dC. \tag{6}$$

After integrating, we find

$$\alpha_1^* = \frac{4}{\sqrt{\pi}} \lambda N_1 \sqrt{R+r_0} \left[1 - \frac{24}{\pi} \frac{r_0}{C^* + r_0 - d} \left(\frac{r_0}{R+r_0}\right)^{1/2} \ln \frac{C^* + r_0}{d}\right]$$

$$\times\left\{\sqrt{\frac{\pi}{2}} \left[\frac{1}{3}\sqrt{C^* + 4r_0 - d} - \frac{\sqrt{2(R+r_0)}}{\pi r_1}(d - 3r_0)\right] \operatorname{erfc}\left(\frac{d}{\sqrt{2}\sigma_1}\right) + \frac{\sqrt{2(R+r_0)}}{\pi r_1} \sigma_1 \exp\left(-\frac{d^2}{2\sigma_1^2}\right)\right\}. \tag{7}$$

The external load is related to the distance d between the surfaces by the equation

$$P_1 = \left(\frac{1}{0.83}\right)^{3/2} \frac{E\sqrt{r_0}}{1-\mu^2} \int\limits_d^{C^*+r_0} \int\limits_0^{B_j} b_j^{3/2} n(C)\, n(b_j)\, dC\, db_j, \tag{8}$$

from which it follows that

$$P_1 = \frac{1}{50}\left(\frac{1}{0.83}\right)^{3/2} \sqrt{\frac{\pi}{2}} \frac{N_1 E(R+r_0)}{1-\mu^2} \left(\frac{C^* + 2r_0 - d}{r_0}\right)^{3/2} \Psi(d, \sigma_1),$$

$$\Psi(d, \sigma_i) = \sigma_i \left[\exp\left(-\frac{d^2}{2\sigma_i^2}\right) - \exp\left(-\frac{\bar{C}^2}{2\sigma_i^2}\right)\right] - \sqrt{\frac{\pi}{2}} (d - r_0) \operatorname{erfc}\left(\frac{d}{\sqrt{2}\sigma_i}\right), \quad \bar{C} = C^* + r_0, \tag{9}$$

and the actual relative area of contact is

$$F_1 = \frac{\pi}{32}\sqrt{\frac{\pi}{2}} \frac{R+r_0}{r_0} N_1 (C^* + 2r_0 - d)\, \Psi(d, \sigma_1). \tag{10}$$

Cylindrical Undulations. In this case

$$n(b_j)\, db_j = \frac{l(R+r_0)\, db_j}{8r_0^2\sqrt{(R+r_0)^2 - (R - C + b_j)^2}}, \tag{11}$$

and the integrated conductivity through the discrete contact points on the individual wave is

$$\alpha_2 = \frac{1}{3\sqrt{2}} \lambda l \frac{B_j}{r_0} \sqrt{\frac{R+r_0}{r_0}}, \tag{12}$$

which depends on the deformation of the wave B_j. Making use of the principle of generalized conductivity, by a process analogous to that of [6] we find that the thermal resistance of the wave is

$$R_2 = \frac{1}{\pi\lambda\sqrt{Al}} \operatorname{arctg} \frac{r_2}{\sqrt{Al}} + \frac{1}{\alpha_2}, \quad r_2 = \frac{1}{2}\sqrt{\frac{1}{n_2}}, \tag{13}$$

where we take the quantity A from the expression (2) as the half-width of the contact strip. It follows from (13) that

$$\alpha_2^* = 4\sqrt{\frac{2}{\pi}} \left[2(R+r_0)\right]^{1/4} \lambda N_2 \sqrt{l} \left[1 - \frac{6r_0}{(R+r_0)^{1/4}} \sqrt{\frac{r_0}{l}}\right.$$

$$\left.\times \left(\frac{2}{C^* + r_0 - d}\right)^{3/4} \ln \frac{C^* + r_0 - d}{r_0}\right] \left\{\sqrt{\frac{\pi}{2}} \left[\frac{1}{5}(C^* + 4r_0 - d)^{1/4}\right.\right.$$

$$-(d-3r_0)\frac{[2(R+r_0)]^{1/4}}{\pi r_2}\sqrt{\frac{l}{C^*+r_0-d}}\Bigg]\operatorname{erfc}\left(\frac{d}{\sqrt{2}\sigma_2}\right)+\frac{[2(R+r_0)]^{1/4}}{\pi r_2}\sqrt{\frac{l}{C^*+r_0-d}}\,\sigma_2\exp\left(-\frac{d^2}{2\sigma_2^2}\right)\Bigg\}. \quad (14)$$

This equation contains the value of the distance between the surfaces, which may be expressed in terms of the load by means of the relation

$$P_2=\frac{1}{12\sqrt{\pi}}\left(\frac{1}{0.83}\right)^{3/2}\frac{N_2El}{1-\mu^2}\left(\frac{R+r_0}{r_0}\right)^{3/2}\Psi(d,\ \sigma_2). \quad (15)$$

Finally, the actual relative area of contact is

$$F_2=\frac{\sqrt{\pi}}{4}\,\frac{N_2(R+r_0)\,l}{r_0}\sqrt{\frac{R+r_0}{C^*+r_0-d}}\,\Psi(d,\ \sigma_2). \quad (16)$$

In the expressions (7) and (14), the term with the minus sign containing a logarithm in the square brackets represents the reduction in the conductivity of the contact between the undulating surfaces, owing to the fact that this contact includes a resistance due to small-scale microscopic concentrations of thermal flux. It follows from (7) and (14) that this component of the contact thermal resistance, which depends on the undulation and roughness parameters, is about two orders of magnitude less than the component due to the large-scale concentration of the lines of thermal flux. This conclusion agrees with the results of [5], where unit contact between two hemispheres was considered.

If heat transfer by radiation and through the intervening medium are characterized by the coefficients α_{rad} and α_{int}, the total coefficient of contact conductivity will be

$$\bar{\alpha}_i=\alpha_i^*+(1-F_i)(\alpha_{rad}+\alpha_{int}),\quad i=1,\ 2.$$

As in the case of rough surfaces [1, 2], the contact thermal resistance of undulating surfaces will be exponential in nature, depending on the load.

If in a contact between undulating surfaces we know what fraction β of the total number of microasperities is plastically deformed, then the results obtained for α_i^* and F_i may easily be generalized, as in [1], to the case of elastic–plastic contact. We shall show how the external load may be calculated in the case of elastic–plastic contact between rough surfaces. (In what follows we shall use the notation of [1].) For the case of plastic deformation, the stress on a microasperity is equal to the hardness H of its material, so that the deformation of the asperity is related to the load P_i applied to the latter by the equation

$$b_i=\frac{P_i}{2\pi RH}.$$

The plastically deformed asperities of the rough surface are subjected to a load equal to

$$P_{pl}=\beta\frac{\sqrt{2\pi}NRH}{\sigma}\int_d^{z^*}(z-d)\exp\left(-\frac{z^2}{2\sigma^2}\right)dz=\beta\sqrt{2\pi}\,NRH\sigma$$

$$\times\left[\exp\left(-\frac{d^2}{2\sigma^2}\right)-\sqrt{\pi}\,\frac{d}{\sqrt{2}\sigma}\operatorname{erfc}\left(\frac{d}{\sqrt{2}\sigma}\right)\right],\quad \beta=\exp\left(-\frac{b^*}{\sigma}\right), \quad (17)$$

so that the total load is equal to

$$\bar{P}=(1-\beta)P+P_{pl}, \quad (18)$$

where P is the load on the elastically deformed asperities. If we know the value of β for the undulating surfaces, then $\bar{P}$ may be calculated by a procedure analogous to that described above, taking account of (3) and (11).

A numerical comparison of the resulting relations and the experimental data will not be carried out here, since the author has no knowledge of the results of any experiments with undulating surfaces.

NOTATION

n_i represents the number of spherical (i = 1) and cylindrical (i = 2) waves which have come into contact with the smooth surface;
N_i represents the total number of corresponding waves per unit area of nominal contact surface;
σ_i^2 is the effective dispersion of wave amplitude distribution;

C	is the wave height;
C^*	is the maximum wave height;
r_0, b_j	are, respectively, the radius of a microscopic asperity projection and the deformation of the latter;
R, B_j	are the radius of a wave and its deformation;
A	is the radius of the large region of contact spots for a spherical wave and the half-width of the contact strip for a cylindrical wave;
λ	is the thermal conductivity of the contact zone;
α_i	is the integrated conductivity through the contact spots of the i-th wave;
R_i	is the total thermal resistance of the i-th wave;
r_i	is the radius of the region ($i = 1$) and the half-width of the strip ($i = 2$) from which the lines of thermal flux contract to the large groups of discrete contact spots;
d	distance between the smooth surface and the standard plane of the undulating surface;
P_i, F_i	are the external load on the contact between the two surfaces with the i-th undulation and the actual relative area of contact between them;
E, μ	are the modulus of elasticity and the Poisson ratio for the material of the asperity;
$2l$	is the unit length of cylindrical wave;
α_i^*	is the conductivity of the contact between two surfaces with similar undulation;
α_{rad}, α_{int}	are the coefficients of heat transfer by radiation and through the intervening medium;
$\bar{\alpha}_i$	is the total coefficient of contact conductivity;
$n(C)dc$	is the Gaussian distribution function for the wave amplitudes.

LITERATURE CITED

1. V. S. Novikov, Inzh. -Fiz. Zh., 19, No. 1 (1970).
2. V. S. Novikov and V. L. Chumakov, in: Collection of Works on Heat Exchange and Thermodynamics [in Russian], Naukova Dumka, Kiev, No. 1 (1970).
3. I. V. Klagel'skii, Friction and Wear [in Russian], Mashinostroenie, Moscow (1968).
4. B. I. Segal and K. A. Semendyaev, Five-Figure Mathematical Tables [in Russian], Fizmatgiz, Moscow (1961).
5. A. Clausing and B. Chao, Teploperedacha, No. 3, 98 (1965).
6. R. Holm, Electrical Contacts [Russian translation], IL (1961).